Application of Domain Decomposition combined Radial Basis Function Collocation Method in Moving Conductor Eddy Current magnetic Problems

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Abstract — Radial basis function (RBF) collocation method, as a newly developed meshless method, shows a great advantage in the computation of transient eddy current magnetic problems with moving conductors. In the former research, the magnetic field was solved by time-domain iteration. One problem is that the coefficient matrix of the RBF governing equations, which needs to be computed in each iteration step, is full. As the number of RBF nodes used in the model increases, the computational capacity will grow rapidly. The domain decomposition method (DDM) is a useful tool to divide the solving domain into several sub-domains and could be conveniently combined with RBF collocation method. This paper first applies DDM combined RBF collocation method to compute transient electromagnetic problems with moving conductors. With this method, the iteration only proceeds in the sub-domain which contains the conductor part. And the magnetic field in the sub-domains without conductors needs to be computed only once before the iteration. Therefore, the size of the coefficient matrix used in the iteration is determined by the number of RBF nodes in the corresponding sub-domain and on the link boundary. An engineering problem is computed to show that the DDM combined RBF method is much more efficient than the normal RBF method.

I. INTRODUCTION

The research on the computation of transient eddy current problems coupled with conductor movement has always been attractive in computational electromagnetics. In [1], time-domain iteration is constructed by radial basis function (RBF) collocation method to compute the convective diffusion equations of the magnetic field. However, the coefficient matrix of the RBF collocation governing equation, which needs to be solved in each iteration step, is full. As the number of RBF node used in the model increases, the computation will become time-consuming.

To reduce the computational capacity of RBF collocation method, one effective way is to make its coefficient matrix of governing equations become a sparse one like the finite element method (FEM). The domain decomposition method (DDM) is a useful tool to divide the solving domain into several sub-domains. And this method has been introduced into RBF collocation method to compute electrostatic problems with complex materials [2]. In the transient eddy current problems, only the magnetic field on the nodes belonging to the conductor area is governed by the convective diffusion equations. The field on the other nodes still satisfies the Poisson equation as static problems. With DDM, the iteration could only proceed in the sub-domain containing the conductor. Therefore, the size of the coefficient matrix used in the iteration is determined by the number of RBF nodes in the corresponding sub-domain and on the link boundary. Through the numerical example, we could see that the computational capacity could be greatly reduced with DDM combined RBF compared with the normal RBF collocation method.

II. RBF COLLOCATION GOVERNING EQUATIONS FOR MOVING CONDUCTOR EDDY CURRENT PROBLEMS

Without loss of generality, consider a 2-D homogeneous and isotropic electromagnetic system in which a nonmagnetic conductor is restricted to move along the y- axis direction with a speed V. The governing equation of the magnetic field in the conductor is:

$$\sigma\mu\frac{\partial A}{\partial t}-\nabla^2 A+\sigma\mu V\frac{\partial A}{\partial y}=\mu J_s \tag{1}$$

where A=$\boldsymbol{A_z}$ is the z-axis component of magnetic vector potential and J_s is the excitation current density. The conductivity σ and the permeability μ are assumed constant. The magnetic field is approximated by RBF as:

$$A(\boldsymbol{x},t)=\sum_{i=1}^{N}a_i(t)\varphi(\|\boldsymbol{x}-\boldsymbol{x}_i\|)=\boldsymbol{\varphi}^T(\boldsymbol{x})\boldsymbol{a}(t) \tag{2}$$

where $\varphi(\|\boldsymbol{x}-\boldsymbol{x}_i\|)$ is the RBF centered at the node with a coordinate $\boldsymbol{x_i}=(x_i,y_i)$, $\|\cdot\|$ means the Euclid norm. $\boldsymbol{a}(t)$ is the unknown coefficient vector and N is the number of RBF nodes. The RBF collocation model in moving coordinate systems could be seen in [1]. The magnetic field is regarded as a superposition of two fields A_s and A_e approximated respectively by separate RBFs in moving coordinate systems as:

$$A=A_s+A_e=\boldsymbol{\varphi}_s{}^T(\boldsymbol{x_s})\boldsymbol{a}_s(t)+\boldsymbol{\varphi}_e{}^T(\boldsymbol{x_e})\boldsymbol{a}_e(t) \tag{3}$$

where the subscript s and e mean the excitation current and eddy current respectively. The iteration scheme to solve (1) could be written as [1]:

$$\boldsymbol{a}_s{}^{k+1}=L_s(\boldsymbol{a}_s{}^k,\boldsymbol{\varphi}_s,J_s{}^k) \tag{4a}$$

$$\boldsymbol{a}_e{}^{k+1}=L_e(\boldsymbol{a}_e{}^k,\boldsymbol{\varphi}_e,\boldsymbol{a}_s{}^k,\boldsymbol{a}_s{}^{k+1},V^k,J_s{}^k)\ ,\quad k=1,2,\cdots \tag{4b}$$

where $L(.)$ refers to a linear operator. In the iteration, A_s is computed as static problems. If we know the variation of J_s, $\boldsymbol{a}_s$ needs to be calculated only once and the results in the following iteration step could be obtained by the linear relationship between $\boldsymbol{a}_s$ and J_s. So the computational capacity of A_s could be ignored. Since A_e needs to be computed in each iteration step, the size of coefficient matrix is mainly determined by its RBF nodes number N_e.

III. DOMAIN DECOMPOSITION METHOD

The main idea of DDM is to divide the solving domain into several sub-domains in which the field could be computed simultaneity [2]. Assuming that the solving domain with its boundary is divided into two parts Ω_1 and Ω_2, Γ is the boundary between them as: $\Gamma = \Omega_1 \cap \Omega_2$. The magnetic field fulfills:

$$L_1(A_1) = g_1 \quad \text{in } \Omega_1 \tag{5a}$$

$$L_2(A_2) = g_2 \quad \text{in } \Omega_2 \tag{5b}$$

$$\partial A_1 / \partial n - \partial A_2 / \partial n = 0 \quad \text{on } \Gamma \tag{5c}$$

where n is the normal direction of Γ. Set N_1 and N_2 RBF nodes in Ω_1 and Ω_2 respectively. Among them, the last N_0 nodes are established on Γ. Then set another N_Γ nodes on Γ. The coordinates of these nodes are denoted by subscripts *1*, *2* and Γ according to their location. Combing DDM with RBF collocation method, equation (5) could be solved as follows:

1. Let $A_1 = A_1' + \lambda_1$, $A_2 = A_2' + \lambda_2$. Using the subscript p to denote the corresponding sub-domains, we have:
   $A_p = A_p' + \lambda_p$ where p=1 or 2.
2. Express A_p' with RBF respectively and their coefficient parameter $\boldsymbol{a_p}$ could be solved from:

$$L_p(\sum_{j=1}^{N_p} a_{pj} \varphi(\| \boldsymbol{x}_i - \boldsymbol{x}_{pj} \|)) = g_p(\boldsymbol{x}_i) \quad i = 1, 2, \cdots N_p - N_0 \tag{6a}$$

$$\sum_{j=1}^{N_p} a_{pj} \varphi(\| \boldsymbol{x}_i - \boldsymbol{x}_{pj} \|) = 0 \quad i = N_p - N_0 + 1, \cdots N_p \tag{6b}$$

3. The component λ relates to the nodes on Γ as:

$$\lambda_p = \sum_{k=1}^{N_\Gamma} a_{\Gamma pk} \psi_{pk}(\boldsymbol{x}, \boldsymbol{x}_{\Gamma k}) = \boldsymbol{\psi}^T \boldsymbol{a_{\Gamma p}} \quad \boldsymbol{x} \in \Omega_p \tag{7}$$

where the function $\psi_{pk}(\boldsymbol{x}, \boldsymbol{x}_{\Gamma k})$ is expressed with the RBFs in Ω_p as:

$$\psi_{pk}(\boldsymbol{x}, \boldsymbol{x}_{\Gamma k}) = \sum_{j=1}^{N_p} \beta_{pj} \varphi(\| \boldsymbol{x} - \boldsymbol{x}_{pj} \|) \quad \boldsymbol{x} \in \Omega_p \tag{8}$$

The coefficient $\boldsymbol{\beta_p}$ is solved by:

$$L_p(\sum_{j=1}^{N_p} \beta_{pj} \varphi(\| \boldsymbol{x}_i - \boldsymbol{x}_{pj} \|)) = 0 \quad i = 1, 2, \cdots N_p - N_0 \tag{9a}$$

$$\sum_{j=1}^{N_p} \beta_{pj} \varphi(\| \boldsymbol{x}_i - \boldsymbol{x}_{pj} \|) = \varphi(\| \boldsymbol{x}_{\Gamma k} - \boldsymbol{x}_{pj} \|) \quad i = N_p - N_0 + 1, \cdots N_p \tag{9b}$$

4. Compute the coefficient $\boldsymbol{a_\Gamma}$ with:

$$\sum_{j=1}^{N_\Gamma} (a_{\Gamma 1j} \frac{\partial \psi_{1j}}{\partial n}(\boldsymbol{x}_i, \boldsymbol{x}_{\Gamma j}) - a_{\Gamma 2j} \frac{\partial \psi_{2j}}{\partial n}(\boldsymbol{x}_i, \boldsymbol{x}_{\Gamma j})) = \sum_{j=1}^{N_2} a_{2j} \frac{\partial \varphi}{\partial n}(\| \boldsymbol{x}_i - \boldsymbol{x}_{2j} \|) - \sum_{j=1}^{N_1} a_{1j} \frac{\partial \varphi}{\partial n}(\| \boldsymbol{x}_i - \boldsymbol{x}_{1j} \|) \quad i = 1, 2, \cdots N_\Gamma \tag{10}$$

5. Compute A_p with:

$$A_p(\boldsymbol{x}) = \boldsymbol{\varphi}^T(\boldsymbol{x}) \boldsymbol{a_p} + \boldsymbol{\psi}^T(\boldsymbol{x}) \boldsymbol{a_{\Gamma p}} \tag{11}$$

The conductor is assumed to be in the sub-domain Ω_1. Applying (6)-(11) to compute the eddy current magnetic field A_e, we need to solve two matrix equations in each iteration steps and the size of them is N_1 and N_Γ. Apparently, if we divide more sub-domains, the computational capacity would be further reduced. Expressing the boundary between them with $\Gamma_1, \Gamma_2, \cdots \Gamma_M$, equation (10) must be applied on all of them and $N_\Gamma = \sum_{m=1}^{M} N_{m\Gamma}$.

IV. NUMERAL EXAMPLE AND CONCLUSION

An electromagnetic launcher system, which could be seen in [3], is analyzed with normal and DDM combined RBF collocation method for compare. The parameters of the RBF models and the computing time are shown in Table I. Six sub-domains are divided in the DDM combined RBF model. Both methods could obtain a proper result compared with the experiment data from [3]. However, from the table we could see that the DDM combined RBF is much more efficient. More details about the model and the numerical results will be presented in the full paper.

TABLE I
INFORMATION OF RBF MODEL AND COMPUTING TIME

	Normal RBF collocation method	DDM combined RBF collocation method
Nodes number used in the iteration	N_e=2047	N_{pe}=308, N_Γ=127
Computing time for one iteration step (unit: s)	31.2	0.7

V. REFERENCES

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